Bottonium mass – evaluation using renormalon cancellation

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We present a method of calculating the bottonium mass $M_\Upsilon(1S) = 2m_b + E_{b\bar{b}}$. The binding energy is separated into the soft and ultrasoft components $E_{b\bar{b}} = E_{b\bar{b}}(s) + E_{b\bar{b}}(us)$ by requiring the reproduction of the correct residue parameter value of the renormalon singularity for the renormalon cancellation in the sum $2m_b + E_{b\bar{b}}(s)$. The Borel resummation is then performed separately for $2m_b$ and $E_{b\bar{b}}(s)$, using the infrared safe $\overline{m}_b$ mass as input. $E_{b\bar{b}}(us)$ is estimated. Comparing the result with the measured value of $M_\Upsilon(1S)$, the extracted value of the quark mass is $\overline{m}_b(\mu = \overline{m}_b) = 4.241 \pm 0.068$ GeV (for the central value $\alpha_s(M_Z) = 0.1180$). This value of $\overline{m}_b$ is close to the earlier values obtained from the QCD spectral sum rules, but lower than from pQCD evaluations without the renormalon structure for heavy quarkonia.

Heavy quarkonia $q\bar{q}$ ($q = b, t$) can be investigated by perturbative methods (pQCD) via effective theories NPQCD [1] and pNRQCD [2] (or: vNRQCD [3]) because of the scale hierarchies of the problem: $m_q > m_q \alpha_s(\mu_s) > m_q \alpha_s^2(\mu_{us}) \gtrsim \Lambda_{\rm QCD}$. Here, m_q is the (pole) mass of the quark, $\mu_s \sim m_q \alpha_s(\mu_s)$ is the soft, and $\mu_{us} \sim m_q \alpha_s^2(\mu_{us})$ the ultrasoft energy. The quarkonium mass is $M_{q\bar{q}} = 2m_q + E_{q\bar{q}}$, where the binding energy consists of the soft and ultrasoft regime contributions: $E_{q\bar{q}} = E_{q\bar{q}}(s) + E_{q\bar{q}}(us)$. A practical problem which appears in the course of evaluation of $M_{q\bar{q}}$ is that the perturbative pole mass has an inherent ambiguity $\delta m_q \sim \Lambda_{\rm QCD}$ (~ 0.1 GeV) due to the infrared (IR) renormalon singularity which appears at the value of the Borel transform variable $b = 1/2$ for $m_q/\overline{m}_q$. Here, $\overline{m}_q$ is the infrared safe (renormalon-free) $\overline{\rm MS}$ mass. However, the static potential $V_{q\bar{q}}(r)$ has a related ambiguity $\delta V_{q\bar{q}}(r) \sim \Lambda_{\rm QCD}$ such that $\delta(2m_q + V_{q\bar{q}}) = 0$, i.e., the renormalon singularity cancels for the combined quantity $2m_q + V_{q\bar{q}}$ [4] (see also [5]). The static potential is a quantity which does not contain ultrasoft regime contributions [6]. Therefore, $E_{q\bar{q}}(s)$ contains the entire $V_{q\bar{q}}$ and kinetic energy effects, the latter are renormalon-free. Thus, the $b = 1/2$ renormalon singularity of $V_{q\bar{q}}$ and $E_{q\bar{q}}(s)$ are equal and hence the singularity must cancel also in the combination $2m_q + E_{\rm q\bar{q}}(s)$

$$\delta \left[2m_q + E_{\rm q\bar{q}}(s) \right] = 0 \ . \tag{1}$$

In principle, $E_{q\bar{q}}(us)$ could be included in this relation. However, in practice, it distorts the cancellation effects since we know these quantities only to a finite order in perturbation expansions [cf. the discussion following Eq. (12)].

One possibility of evaluating the bottonium ground state $\Upsilon(1S)$ mass is to use an infrared-safe (renormalon-free) quark mass ($\overline{m}_b$, $m_b^{\rm RS}$, etc.) and a common couplant $a(\mu) = \alpha_s(\mu)/\pi$ as inputs in the evaluation of the available truncated perturbation expansion (TPS) for $M_\Upsilon(1S) = 2m_b + E_{b\bar{b}}$, in order to avoid the $b = 1/2$ renormalon (divergence) problems throughout [7], and then extract the value of $\overline{m}_b$ from the measured value $M_\Upsilon(1S) = 9460$ MeV.

Another possibility is to evaluate $E_{b\bar{b}}$ in terms of the pole mass m_b and of $a(\mu)$, taking into account the $b = 1/2$ singularity of $E_{b\bar{b}}$ (using, e.g., the Principal Value [PV] prescription in the Borel integration), and adding $2m_b$ to $E_{b\bar{b}}$. From $M_\Upsilon(1S) = 2m_b^{\rm (PV)} + E_{b\bar{b}}(m_b^{\rm (PV)})$, the PV-value of the pole mass m_b is then extracted, and subsequently the value of $\overline{m}_b$ (via PV Borel integration prescription). This is the approach of Ref. [8].

Our approach [9] follows to a significant de-

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gree the latter approach, but with some important modifications:

1. The input parameter is the renormalon-free mass $\overline{m}_b \equiv \overline{m}_b(\mu = \overline{m}_b)$ (and, of course, the QCD couplant $a(\mu)$). The pole mass $m_b = m_b(\overline{m}_b; \mu_m)$ is evaluated via Borel integration, accounting for the $b = 1/2$ singularity, and using a hard renormalization scale $\mu_m \sim m_b$. The residue parameter N_m of the $b = 1/2$ singularity is evaluated from the available TPS for $m_b/\overline{m}_b$.

2. On the basis of the knowledge of N_m, we separate the binding energy into the soft (s) and ultrasoft (us) regime contributions: $E_{b\bar{b}} = E_{b\bar{b}}(s; \mu_f) + E_{b\bar{b}}(us; \mu_f)$, where the s-us factorization scale μ_f parametrizes the separation. The separation is performed by accounting for the renormalon cancellation in the sum $2m_b + E_{b\bar{b}}(s)$: $N_m(m_b) = N_m(E_{b\bar{b}}(s; \mu_f))$. The latter relation fixes μ_f and thus the separation.

3. The soft binding energy $E_{b\bar{b}}(s; \mu_f)$ is then evaluated via Borel integration, accounting for the $b = 1/2$ singularity (using the same prescription, e.g. PV, as for m_b), and using a soft renormalization scale $\mu_s \sim m_b \alpha_s$.

4. The value of the ultrasoft part $E_{b\bar{b}}(us; \mu_f)$ is estimated.

5. From $2m_b + E_{b\bar{b}}(s) + E_{b\bar{b}}(us) = M_\Upsilon(1S)$, the value of $\overline{m}_b$ is extracted.

For details, we refer to Ref. [9].

1. Evaluation of m_b and N_m

This part has been performed mostly in Refs. [7,8,10]. The pole mass is known to NLO:

$$S \equiv \frac{m_b}{\overline{m}_b} - 1 = \frac{4}{3} a(\mu_m) \sum_{j=0}^{\infty} a^j(\mu_m) r_j(\mu_m) \ , \qquad (2)$$

where r_1 and r_2 are known coefficients ($r_0 = 1$), e.g., in the $\overline{\text{MS}}$ scheme, and they depend on $(\mu_m/\overline{m}_b)$; $\mu_m \sim \overline{m}_b$. The Borel transform is

$$B_S(b) = \frac{4}{3}\left[1 + \frac{r_1}{1!\beta_0} b + \frac{r_2}{2!\beta_0^2} b^2 + \mathcal{O}(b^3)\right] \qquad (3)$$

$$= \frac{N_m \pi \mu_m}{\overline{m}_b (1-2b)^{1+\nu}} \sum_{k=0}^{\infty} \tilde{c}_k (1-2b)^k + B_S^{(\text{an.})}(b), \ (4)$$

where $\beta_0 = (11 - 2n_f/3)/4$, $\beta_1 = (102 - 38n_f/3)/16$, $\nu = \beta_1/(2\beta_0^2)$ ($n_f = 4$); $\tilde{c}_0 = 1$ and the next three coefficients $\tilde{c}_k$ are known ([7] for $k = 1, 2$; [9] for $k = 3$). $B_S^{(\text{an.})}(b)$ is the analytic part in the bilocal expansion (4) [8], and it is known up to $\sim b^2$. The residue parameter N_m in Eq. (4) can be obtained with high precision [7,8,10]

$$N_m = \frac{\overline{m}_q}{\mu_m} \frac{1}{\pi} R_S(b = 1/2) \ , \qquad (5)$$

where, according to (4)

$$R_S(b; \mu_m) \equiv (1 - 2b)^{1+\nu} B_S(b; \mu_m) \ . \qquad (6)$$

Applying the Padé P[1/1] to the known NNLO TPS of $R_S(b)$ then gives

$$N_m(n_f = 4) = 0.555 \pm 0.020 \ . \qquad (7)$$

The pole mass m_b, with $\overline{m}_b$ and $a(\mu_m)$ as input, can now be evaluated by Borel integration using the bilocal expression (4)

$$S(b) = \frac{1}{\beta_0} \text{Re} \int db \exp\left(-\frac{b}{\beta_0 a(\mu_m)}\right) B_S(b; \mu_m) \ , (8)$$

where the integration path can be taken along a ray in the first or fourth quadrant (the generalized PV prescription [11,12,13]).

2. Separation

The TPS of the binding energy $E_{b\bar{b}}$

$$E_{b\bar{b}} = -\frac{4\pi^2}{9} \overline{m}_b a^2 \sum_{k=0}^{\infty} a^k f_k \ , \qquad (9)$$

is known to the impressive order $\mathcal{O}(m_b a^5)$ [14,15,16,17,18,19,20], i.e., in Eq. (9) f_k ($k = 1, 2, 3$) are known ($f_0 = 1$). The renormalization scale used in expansion (9) should be soft ($\mu_s \sim m_b \alpha_s$) or lower. The ultrasoft contributions appear for the first time at $\sim m_b a^5$ [19,20], i.e., $f_3 = f_3(s) + f_3(us)$. The us coefficient can be written as [9]: $f_3(us)/\pi^3 = 27.5 + 7.1 \ln \alpha_s(\mu_s) - 14.2 \ln \kappa$, where $\kappa \sim 1$ is the parameter of the s-us factorization scale μ_f: $\mu_f = \kappa m_b \alpha_s(\mu_s)^{3/2}$. It can be fixed by

the requirement of the renormalon cancellation in $2m_b + E_{b\bar{b}}(s)$:

$$N_m = \frac{2\pi}{9} \frac{\overline{m}_b a(\mu_s)}{\mu_s} R_{F(s)}(b;\mu_s;\mu_f)\big|_{b=1/2} \ , \qquad (10)$$

where, in analogy with R_S of (6)

$$R_{F(s)}(b;\mu_s;\mu_f) = (1-2b)^{1+\nu} B_{F(s)}(b;\mu_s;\mu_f) \ , (11)$$

and $B_{F(s)}$ is the Borel transform of the quantity $F(s) = -(9/(4\pi^2))E_{b\bar{b}}(s)/(\overline{m}_b a(\mu_s))$ [in analogy with S of (2)]. Since now the TPS of $R_{F(s)}$ is known to $\sim b^3$, the Padé P[2/1](b) thereof can be taken; using then the value (7) of N_m, the renormalon cancellation condition (10) gives numerically the s-us separation parameter

$$\kappa = 0.59 \pm 0.19 \ . \qquad (12)$$

It was possible to obtain the value of μ_f ($\Leftrightarrow \kappa$) because the dependence on μ_f in N_m of Eq. (10) was taken (and is known) only to the leading order, the ultrasoft part was excluded, and N_m is well-known (7). This is similar to the scale-fixing in the effective charge (ECH) method [21]. If the ultrasoft contributions are included in Eq. (10), the value (7) of N_m cannot be reproduced.

3. Evaluation of the soft contributions

Knowing now the expansion of $F(s) = -(9/(4\pi^2))E_{b\bar{b}}(s)/(\overline{m}_b a(\mu_s))$ up to $\sim a^4$, the Borel transform of this quantity can be constructed, e.g., with the approach of the "σ-regularized" bilocal expansion [9], which is a generalization of the bilocal expansion (4)

$$B_{F(s)}(b) = \frac{9 N_m \mu_s}{2\pi \overline{m}_b a(\mu_s)(1-2b)^{1+\nu}} \left[\sum_{k=0}^{\infty} \tilde{\mathcal{C}}_k (1-2b)^k \right] \times \exp\left[-\frac{1}{8\sigma^2}(1-2b)^2 \right] + B_{F(s)}^{(\mathrm{an.})}(b) \ . \qquad (13)$$

The exponential was introduced in order to suppress the renormalon part away from $b \approx 1/2$. The first four coefficients $\tilde{\mathcal{C}}_k$ are known ($\tilde{\mathcal{C}}_0 = 1$), and the analytic part is known now up to $\sim b^3$. The analytic part we can evaluate either as TPS or as Padé P[2/1](b). The requirement of the absence of the pole around $b = 1/2$ in that part, and the independence (weak dependence) on the renormalization scale μ_s for the Borel-resummed result $E_{b\bar{b}}(s)$, lead us to fix the σ parameter to the values $\sigma = 0.36 \pm 0.03$. The Borel integration is performed as in Eq. (8), with the ray (PV) path prescription taken.

4. Estimate of the ultrasoft contribution

The ultrasoft part of the energy is known only to the leading order ($\sim m_b a^5$)

$$\begin{aligned} E_{b\bar{b}}(us)^{(\mathrm{p})} &\approx -\frac{4}{9}\overline{m}_q \pi^2 f_3(us) a^5(\mu_{us}) \\ &\approx (-150 \pm 100) \ \mathrm{MeV} \ . \end{aligned} \qquad (14)$$

Here, $f_3(us;\mu_f)$ was determined in Sec. 2; the ultrasoft renormalization scale μ_{us} should be $\sim \alpha_s^2 m_b$, but was taken numerically to be higher, in the soft regime ($\mu \approx$ 1.5-2.0 GeV $\Rightarrow$ $\alpha_s(\mu) \approx$ 0.30-0.35), because perturbative QCD does not allow a running to very low scales. The bottom mass value was taken $\overline{m}_b = 4.2$ GeV. The non-perturbative contribution comes primarily from the gluonic condensate and gives $E_{b\bar{b}}(us)^{(\mathrm{np})} \approx 50 \pm 35$ MeV if the gluon condensate values $\langle (\alpha_s/\pi) G^2 \rangle = 0.009 \pm 0.007 \ \mathrm{GeV}^4$ [22] are taken. This then results in the following estimate of the ultrasoft contributions to the binding energy

$$E_{b\bar{b}}(us)^{(\mathrm{p+np})} \approx (-100 \pm 106) \ \mathrm{MeV} \ . \qquad (15)$$

In addition, there are contributions to the $\Upsilon(1S)$ mass due to the nonzero mass of the charm quark [23] $\delta M_{\Upsilon}(1S, m_c \neq 0) \approx 25 \pm 10$ MeV.

5. Extraction of the mass $\overline{m}_b$

Adding together the Borel-resummed values $2m_b$, $E_{b\bar{b}}(s)$ and $E_{b\bar{b}}(us)$, requiring the reproduction of the measured mass value $M_{\Upsilon}(1S)$ (with the mentioned $m_c \neq 0$ effect subtracted), we extract the following value for the mass $\overline{m}_b \equiv \overline{m}_b(\mu{=}\overline{m}_b)$:

$$\overline{m}_b(\overline{m}_b) = 4.241 \pm 0.068 \ \mathrm{GeV} \ , \qquad (16)$$

when the QCD coupling value is taken as $\alpha_s(M_Z) = 0.1180 \pm 0.0015$. The major source of uncertainty in the result (16) is the uncertainty from the ultrasoft contributions (15) ($\pm$0.049 GeV). The other appreciable uncertainties are

from the ambiguity of the soft renormalization scale $\mu_s = 3 \pm 1$ GeV (± 0.013 GeV) and of $\alpha_s(M_Z) = 0.1180 \pm 0.0015$ (± 0.013 GeV). If the central value of the gluon condensate $\langle (\alpha_s/\pi) G^2 \rangle$ is increased from 0.009 [22] to 0.024 GeV^4 (used in [24,25]), the central value (16) decreases to $\overline{m}_b(\overline{m}_b) = 4.204$ GeV. This is close to the values of QCD spectral sum rule calculations which gave central values $\overline{m}_b(\overline{m}_b) = 4.20$ GeV [24]; 4.24 GeV [25]; and $\overline{m}_b(m_b) = 4.23$ GeV [26] (Ref. [26] uses central condensate value $\langle (\alpha_s/\pi) G^2 \rangle = 0.019$). The TPS evaluation of $M_\Upsilon(1S)$, without accounting for the renormalon problem, extracts higher central values $\overline{m}_b(\overline{m}_b) = 4.349$ [20].

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